Asserting Controller Robustness and Safety of Unmanned Aircraft Systems

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• Funding: NSF grant # CMMI-1351640 and NAVAIR contract # N00421-16-2-0001
Outline

• Goals and Objectives
• Robust control and Integral Quadratic Constraint (IQC) theory
• Unmanned Aerial Systems (UAS) and IQC analysis framework
• Results
• Extensions
Goals

- UAS dynamics are highly nonlinear, and sensitive to model uncertainties and external disturbances.

- Despite nonlinearities, uncertainties, and disturbances, we want to assert if a given control law will:
  1. Stabilize the UAS
  2. Yield good performance
  3. Maintain safe behavior
Objectives

• Characterize and quantify the UAS uncertainties in a manner amenable to IQC analysis

• Apply IQC analysis tools to access the robustness of the flight control system against the considered types and regions of uncertainties and disturbances

• Use analysis results to guide design of controllers and system properties

• Validate through extensive simulations and flight tests
Algorithmic Level Validation

Model → Simplify

Reasonably accurate nonlinear model → Simplify

Remaining uncertainty of physical system

Simplified Plant Model → Synthesize

Ignored dynamics and uncertainties

Uncertainty analysis (for specific uncertainty regions)

Requirements achieved?

Discrete-time Controller

Validated Control System

No

Yes
Robust Control

- $M$ is a linear dynamic system (e.g. $x(k + 1) = Ax(k) + Bw(k)$…)
- $w$ is a disturbance signal (e.g. wind/noise)
- $\bar{z}$ is the performance output (e.g. position error)
- $w$ belongs to the signal set $\mathcal{W} \subset \ell_2$
  - $\|w\|_{\ell_2}^2 = \sum_{k=0}^{\infty} w(k)^T w(k) < \infty$ (energy of signal $w$)
  - $\mathcal{W}$ is used to better characterize the disturbances
- The “size” of $M$ is defined by the $\mathcal{W}$-to-$\ell_2$-induced norm
  - $\|M\|_{\mathcal{W} \to \ell_2} = \sup_{w \in \mathcal{W}} \frac{\|Mw\|_{\ell_2}}{\|w\|_{\ell_2}}$
Robust Control

- Uncertainties are incorporated with the $\Delta$ block

$$\Delta = \begin{bmatrix} \Delta_1 & \Delta_2 \end{bmatrix} \in \Delta$$

- Robust stability:
  - $w \mapsto \tilde{z}$ is causal and maps $\ell_2$ to $\ell_2$ for all $\Delta \in \Delta$

- Robust $\mathcal{W}$-to-$\ell_2$-gain performance level $\gamma$:
  - robustly stable + $\sup_{\Delta \in \Delta} \|w \mapsto \tilde{z}\|_{w \mapsto \ell_2} < \gamma$
Integral Quadratic Constraints

- Integral Quadratic Constraint (IQC) theory provides an upper bound $\gamma$
  - Expansive library expressing different uncertainty groups (nonlinearities, time-varying, dynamic, etc.)
  - Allow limiting disturbances to a specified signal set $\mathcal{W} \subset \ell_2$
  - Unifying approach
  - Provides sufficient condition expressed as a linear matrix inequality

$$\begin{align*}
\text{minimize} & \quad c^T x \\
\text{subject to} & \quad F_0 + x_1 F_1 + x_2 F_2 + \cdots + x_n F_n \preceq 0
\end{align*}$$
Unmanned Aerial System

- Uncertainties and nonlinearities in UAS are subdivided into various groups.
Aerodynamic Block

- Aerodynamic equations are nonlinear

\[
F_x = \frac{1}{2} C_x \rho V_a^2 S \\
C_x = C_{x_0} + C_{x_\alpha} \alpha + C_{x_\delta T} \delta_T + \frac{C_{x_T} 2T}{\rho S V_a^2}
\]

\[
F_y = \frac{1}{2} C_y \rho V_a^2 S \\
C_y = C_{y_0} + C_{y_\beta} \beta + C_{y_\delta A} \delta_A + C_{y_\delta R} \delta_R + \frac{b \left( C_{y_p} p + C_{y_r} r \right)}{2V_a}
\]

\[
F_z = \frac{1}{2} C_z \rho V_a^2 S \\
C_z = C_{z_0} + C_{z_\alpha} \alpha + C_{z_\delta E} \delta_E + \frac{C_{z_q} q \bar{c}}{2V_a} + \frac{C_{z_T} 2T}{\rho S V_a^2}
\]

\[
M_l = \frac{1}{2} C_l \rho V_a^2 S b \\
C_l = C_{l_0} + C_{l_\beta} \beta + C_{l_\delta A} \delta_A + C_{l_\delta R} \delta_R + \frac{b \left( C_{l_p} p + C_{l_r} r \right)}{2V_a}
\]

\[
M_m = \frac{1}{2} C_m \rho V_a^2 S \bar{c} \\
C_m = C_{m_0} + C_{m_\alpha} \alpha + C_{m_\delta E} \delta_E + \frac{C_{m_q} q \bar{c}}{2V_a}
\]

\[
M_n = \frac{1}{2} C_n \rho V_a^2 S b \\
C_n = C_{n_0} + C_{n_\beta} \beta + C_{n_\delta A} \delta_A + C_{n_\delta R} \delta_R + \frac{b \left( C_{n_p} p + C_{n_r} r \right)}{2V_a}
\]

- Linearize aerodynamic model as: \( \bar{C}_i(\alpha, \beta, \delta_A, \delta_E, \delta_R, \delta_T, T, p, q, r, V_a) \)

- Model as \( C_i = \bar{C}_i(\alpha, \beta, \delta_A, \delta_E, \delta_R, \delta_T, T, p, q, r, V_a) + \Delta C_i \) for \( i = x, y, z, l, m, n \)
Aerodynamic Block

- $\Delta C_i$ is obtained from flight tests

- $\Delta C_i$, for $i = x, y, z, l, m, n$, is modeled as a bounded, rate-bounded, static, linear, time-varying (RB-SLTV) operator
Aerodynamic Block

- To reduce conservatism, “outliers” are detected and removed
- Technique used is based on ellipsoidal peeling
Control Block

Wind/Turbulence

Linear Aerodynamic Model with Static LTV Perturbations

Dynamic LTI Pert.

Sensors

Actuator & Propulsion Models

Controller

Noise

Saturation & Time Delay

Dynamic LTI Perturbations
Control Block

- Servos are modeled as 2\textsuperscript{nd} order systems, motor as static feedthrough

\[ G_{srv}^c(s) = \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} \]

- Errors in servo model are determined using frequency response data

\[ G_{srv, i}(z) = G_{srv}(z) + \Delta_i(z) \text{ for } i = E, A, R \]

\[ \Delta_i(z), \text{ for } i = E, A, R, T, \text{ is a bounded, dynamic, linear, time-invariant (DLTI) operator} \]
Control Block

- Actuator saturation is a nonlinearity
- Can be expressed as $\text{sat}(u) \approx u - \Delta_\sigma u$, with $\Delta_\sigma \in [0, \sigma_{\text{max}}]$

- $\sigma_{\text{max}}$ represents the max saturation considered in analysis (10%)
- Characterization accurately covers desired saturation region
- Conservatively describes the unsaturated region
- $\Delta_\sigma_i$, for $i = E, A, R, T$, is a bounded, RB-SLTV operator
• Data transmissions and computations incur time delays

• Partial time-step and time-varying in nature

• Previous IQC multipliers treating multi time-step delays are overly conservative

• The conservative uncertainty characterization for saturation reasonably encapsulates partial time-step delays
Dynamic Block

Wind/Turbulence

Linear Aerodynamic Model with Static LTV Perturbations

Saturation & Time Delay

Linear Dynamic Model with Static LTI Perturbations (cg/mass/inertia)

Actuator & Propulsion Models

Dynamic LTI Perturbations

Controller

Sensors

Noise

$W^{-1}$ Dynamic LTI Pert.
Dynamic Block

- Standard 6 DOF equations of motion are nonlinear

\[
\begin{align*}
\dot{p} &= \frac{M_l + (I_y - I_z)qr}{I_x} \\
\dot{q} &= \frac{M_m + (I_z - I_x)pq}{I_y} \\
\dot{r} &= \frac{M_n + (I_x - I_y)pq}{I_z} \\
\dot{u} &= -qw + rv + \frac{F_x}{m} - g \sin \theta \\
\dot{v} &= -ru + pw + \frac{F_y}{m} - g \cos \theta \sin \phi \\
\dot{w} &= -pv + qu + \frac{F_z}{m} - g \cos \theta \cos \phi \\
\end{align*}
\]

Linearize

\[
\begin{align*}
\dot{x} &= A\bar{x} + B \begin{bmatrix} \bar{F} \\ \bar{M} \end{bmatrix} \\
\bar{y} &= C\bar{x} + D \begin{bmatrix} \bar{F} \\ \bar{M} \end{bmatrix}
\end{align*}
\]

- Incorporate a perturbation in the linearized system to reintroduce the effect of nonlinearities to some extent
A DLTI operator is used to address nonlinearities

$W$ is a weighting matrix to normalize scales of states

The perturbation $(\Delta_N)$ is a bounded, DLTI operator
Dynamic Block

- Uncertainty also in mass, center of gravity, and moments of inertia
- Ex. UAS fuel consumption changes mass, CG, and moments of inertia
- 6 DOF equations of motion are more complicated (Bacon and Gregory, 2007)
- Error in CG couples forces and moments and creates very large LFT
- Resultant LFT is too large for conducting IQC analysis
- Using coprime factors reduction, the uncertain system can be reduced to computationally manageable LFT
- Model reduction introduces an error into the system (characterized by $\Delta_{ME}$)
- $\Delta_m, \Delta_{CGx}, \Delta_{CGz}, \Delta_{Ix}, \Delta_{Iy}, \Delta_{Iz}$ are bounded, static, linear, time-invariant (SLTI) operators
- $\Delta_{ME}$ is a bounded, DLTI operator
UAS IQC Framework Overview

Wind/Turbulence

Linear Aerodynamic Model with Static LTV Perturbations

Linear Dynamic Model with Static LTI Pert. (cg/mass/inertia)

Dynamic LTI Perturbations

Sensors

Actuator & Propulsion Models

Controller

Noise
UAS IQC Framework Overview

Wind/Turbulence

Linear Aerodynamic Model
\[ \Delta C_x, \Delta C_y, \Delta C_z, \Delta C_l, \Delta C_m, \Delta C_n \]

Linear Dynamic Model
\[ \Delta m, \Delta C_{Gx}, \Delta C_{Gz}, \Delta I_x, \Delta I_y, \Delta I_z \]

Sensors

\[ W^{-1} \Delta N W \]

Controller

\[ [\Delta E, \Delta A, \Delta R, \Delta T] \]

Actuator & Propulsion Models

\[ \Delta \sigma_E', \Delta \sigma_A', \Delta \sigma_R, \Delta \sigma_T \]
### UAS IQC Framework Overview

#### Unc. Type Bounds

<table>
<thead>
<tr>
<th>Unc.</th>
<th>Type</th>
<th>Bounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_{C_x}$</td>
<td>1 x 1 RB-SLTV</td>
<td>$-0.054 \leq \Delta_{C_x}(k) \leq 0.026$ $-0.014 \leq \Delta_{C_x}(k+1) - \Delta_{C_x}(k) \leq 0.006$</td>
</tr>
<tr>
<td>$\Delta_{C_y}$</td>
<td>1 x 1 RB-SLTV</td>
<td>$-0.045 \leq \Delta_{C_y}(k) \leq 0.037$ $-0.012 \leq \Delta_{C_y}(k+1) - \Delta_{C_y}(k) \leq 0.01$</td>
</tr>
<tr>
<td>$\Delta_{C_z}$</td>
<td>1 x 1 RB-SLTV</td>
<td>$-0.113 \leq \Delta_{C_z}(k) \leq 0.119$ $-0.042 \leq \Delta_{C_z}(k+1) - \Delta_{C_z}(k) \leq 0.024$</td>
</tr>
<tr>
<td>$\Delta_{C_l}$</td>
<td>1 x 1 RB-SLTV</td>
<td>$-0.022 \leq \Delta_{C_l}(k) \leq 0.026$ $-0.012 \leq \Delta_{C_l}(k+1) - \Delta_{C_l}(k) \leq 0.009$</td>
</tr>
<tr>
<td>$\Delta_{C_m}$</td>
<td>1 x 1 RB-SLTV</td>
<td>$-0.120 \leq \Delta_{C_m}(k) \leq 0.125$ $-0.046 \leq \Delta_{C_m}(k+1) - \Delta_{C_m}(k) \leq 0.035$</td>
</tr>
<tr>
<td>$\Delta_{C_n}$</td>
<td>1 x 1 RB-SLTV</td>
<td>$-0.006 \leq \Delta_{C_n}(k) \leq 0.006$ $-0.002 \leq \Delta_{C_n}(k+1) - \Delta_{C_n}(k) \leq 0.003$</td>
</tr>
<tr>
<td>$\Delta_{\sigma_E}$</td>
<td>1 x 1 RB-SLTV</td>
<td>$0 \leq \Delta_{\sigma_E}(k) \leq 0.1$ $-0.1 \leq \Delta_{\sigma_E}(k+1) - \Delta_{\sigma_E}(k) \leq 0.1$</td>
</tr>
<tr>
<td>$\Delta_{\sigma_A}$</td>
<td>1 x 1 RB-SLTV</td>
<td>$0 \leq \Delta_{\sigma_A}(k) \leq 0.1$ $-0.1 \leq \Delta_{\sigma_A}(k+1) - \Delta_{\sigma_A}(k) \leq 0.1$</td>
</tr>
<tr>
<td>$\Delta_{\sigma_R}$</td>
<td>1 x 1 RB-SLTV</td>
<td>$0 \leq \Delta_{\sigma_R}(k) \leq 0.1$ $-0.1 \leq \Delta_{\sigma_R}(k+1) - \Delta_{\sigma_R}(k) \leq 0.1$</td>
</tr>
<tr>
<td>$\Delta_{\sigma_T}$</td>
<td>1 x 1 RB-SLTV</td>
<td>$0 \leq \Delta_{\sigma_T}(k) \leq 0.1$ $-0.1 \leq \Delta_{\sigma_T}(k+1) - \Delta_{\sigma_T}(k) \leq 0.1$</td>
</tr>
<tr>
<td>$\Delta_E$</td>
<td>1 x 1 DLTI</td>
<td>$|\Delta_E|_\infty \leq 0.05$</td>
</tr>
<tr>
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<td>1 x 1 DLTI</td>
<td>$|\Delta_R|_\infty \leq 0.05$</td>
</tr>
<tr>
<td>$\Delta_T$</td>
<td>1 x 1 DLTI</td>
<td>$|\Delta_T|_\infty \leq 0.2$</td>
</tr>
</tbody>
</table>

- An appropriate IQC multiplier is also used to characterize sensor noise
Results

• Using MATLAB, the previous framework produces uncertain UAS model
• Uncertainties are scaled with $\epsilon \in [0,1]$

![Diagram of uncertain system](image)

• IQC analysis is conducted by solving the associated semi-definite program
Results (Sensitivities)

• Given a controller, IQC analysis is conducted on uncertain UAS

• Analysis done on separate and combined groups
• Reveals sensitivities to uncertainties
• % Degradation of performance increases nonlinearly
Results (comparison)

• Comparing one controller against another

• Demonstrates improved robust performance level AND reduced sensitivity to uncertainties
Results (tuning)

- Vary controller design parameters to iteratively find controller which yields locally optimal robust performance levels
- Example controller design parameters:
  - PID: $K_P, K_I, K_D$
  - LQR: $Q$ and $R$ matrices
  - $H_\infty$: penalty weights $c_i$ for a given structure of performance output
    \[
    \bar{z}_{syn} = [c_1\bar{p}, c_2\bar{q}, c_3\bar{r}, c_4\bar{\Theta}, c_5\bar{\Phi}, c_6\bar{\Theta}, c_7\bar{\Psi}, c_8\bar{X}, c_9\bar{Y}, c_{10}\bar{h}, c_{11}\bar{E}, c_{12}\bar{A}, c_{13}\bar{R}, c_{14}\bar{T}]
    \]
- Approach:
  - Select number of design parameters $N$ and initial parameter values
  - Loop:
    - Synthesize controller
    - Estimate gradient of “IQC function of the design parameters”
    - Calculate direction of descent
    - Obtain new parameter values by stepping in descent direction
Results (tuning)

$K_D$

$K_P$

$\gamma$

Final controller

Initial controller
Results (tuning)

- Tuning routine starts with bad controller, ends with good controller
- Simulated uncertain UAS to validate IQC analysis claims
- Conducted flight tests for further validation
- Confirms that IQC analysis can qualitatively compare and tune controllers
Results (tuning)
• $\gamma^2$ bounds the scaling between the disturbance and the error signals’ energy
• “Gain between energies” is not an intuitive metric
• More useful in safety assurance: bounding the error at a given time instant
• Recent work enables IQC analysis to produce such guarantees
  – (Fry, Farhood, Seiler 2017)
Questions?
References

• Preliminary work in: